Random wavefunctions and percolation

2007 J. Phys. A: Math. Theor. 40 14033

(http://iopscience.iop.org/1751-8121/40/47/001)

J. Phys. A: Math. Theor. **40** (2007) 14033–14043 doi:10.1088/1751-8113/40/47/001

Random wavefunctions and percolation

E Bogomolny and C Schmit[1]

CNRS, Université Paris-Sud, Laboratoire de Physique Théorique et Modèles Statistiques, 91405 Orsay Cedex, France

E-mail: eugene.bogomolny@lptms.u-psud.fr

Received 4 September 2007, in final form 10 October 2007
Published 6 November 2007
Online at stacks.iop.org/JPhysA/40/14033

Abstract
Recently it was conjectured that nodal domains of random wavefunctions are adequately described by critical percolation theory. In this paper we strengthen this conjecture in two respects. First, we show that, though wavefunction correlations decay slowly, a careful use of Harris' criterion confirms that these correlations are unessential and nodal domains of random wavefunctions belong to the same universality class as non-correlated critical percolation. Second, we argue that level domains of random wavefunctions are described by the non-critical percolation model.

PACS numbers: 05.045.a, 05.40.a, 03.65.w

1. Introduction

In 1977 Berry [1] conjectured that wavefunctions of generic chaotic systems, $\Psi(\vec{x})$, can statistically be written as random superposition of a complete set of functions, $\psi_m(\vec{x})$,

$$\Psi(\vec{x}) = \sum_m C_m \psi_m(\vec{x}), \tag{1}$$

where the coefficients C_m are independent identically distributed random variables with zero mean and variance obtained from normalization. In particular, any wavefunction of a two-dimensional billiard obeys the Helmholtz equation with energy $E = k^2$

$$(\Delta + E)\Psi(x, y) = 0 \tag{2}$$

and can be represented as a formal sum

$$\Psi(x, y) = \sum_m C_m J_{|m|}(kr)\, \mathrm{e}^{\mathrm{i}m\phi}, \tag{3}$$

where r and ϕ are polar coordinates and $J_m(r)$ are the usual Bessel functions. For problems without magnetic field wavefunctions are real and $C_m = C^*_{-m}$.

[1] Deceased.

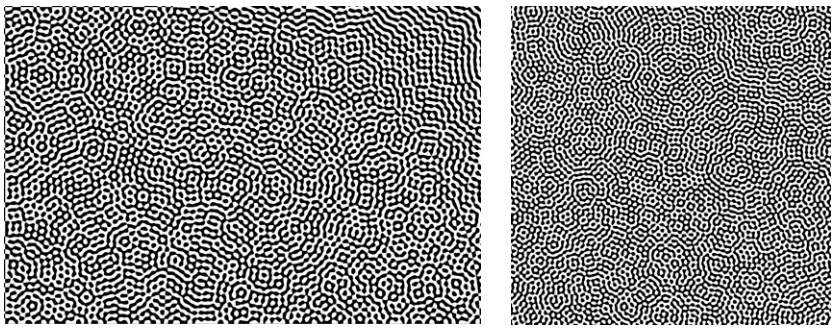

Figure 1. Left: nodal domains of the eigenfunction of a quarter of the stadium with energy $E = 10092.029$. Right: nodal domains of a random wavefunction (3) with $k = 100$.

Consider a chaotic quantum system like the stadium billiard with Dirichlet boundary conditions and let us calculate (numerically) a large number of eigenfunctions with energies close to k^2. Each eigenfunction gives a well-defined set of coefficients, C_m in expansion (3) and it is of interest to know mean values of different functions of these coefficients over the whole ensemble of eigenfunctions. Berry's conjecture means that in the semiclassical limit $k \to \infty$ the result will be the same as if the coefficients C_m would be independent Gaussian random variables with

$$\langle C_m \rangle = 0 \qquad \text{and} \qquad \langle C_m C_n^* \rangle = \sigma^2 \delta_{mn}. \tag{4}$$

Though Berry's conjecture is one of the oldest conjectures in quantum chaos, it has not yet been proved rigorously. Numerically, it works very well. In figure 1, we present two pictures, the left one is a true eigenfunction of a quarter of the stadium with area equals 4π and the right is a random realization of (3) with approximately the same energy. In these figures black and white regions represent nodal domains where the function is, respectively, positive and negative. The two figures look very similar, and the left figure was magnified in order to distinguish better the circular arc of the stadium billiard where the true wavefunction is zero.

In 2002 Blum, Gnutzmann and Smilansky attracted wide attention to such type of pictures by initiating the investigation of nodal domains of wavefunctions for different systems [2]. In [3], it was argued that nodal domains of chaotic wavefunctions (like in figure 1) can adequately be described by a critical percolation model. This conjecture gives a very detailed description of nodal domains and a large number of interesting (global) quantities can directly be transposed from percolation results (see e.g. [5]).

In particular, it was shown in [3][2] that the total number of connected nodal domains for random wavefunctions has Gaussian distribution with a mean value, $\bar{n}(E)$, and variance, $\bar{\sigma}^2(E)$, given by the following expressions:

$$\lim_{E\to\infty} \frac{\bar{n}(E)}{\bar{N}(E)} = \frac{3\sqrt{3}-5}{\pi} \approx 0.0624 \tag{5}$$

and

$$\lim_{E\to\infty} \frac{\bar{\sigma}^2(E)}{\bar{N}(E)} = \frac{18}{\pi^2} + \frac{4\sqrt{3}}{\pi} - \frac{25}{2\pi} \approx 0.0502. \tag{6}$$

Here $\bar{N}(E)$ is the mean number of eigenvalues with energy less than E. For billiards, $\bar{N}(E) \approx Ak^2/4\pi$, where A is the billiard area.

[2] Recently we became aware that similar formulae for the two-dimensional percolation have been obtained by a different method in [4].

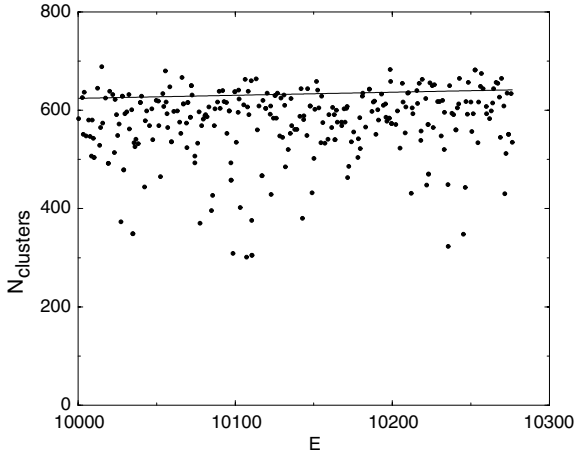

Figure 2. Number of nodal clusters for stadium eigenfunctions versus energy. Solid line is the asymptotic prediction (5).

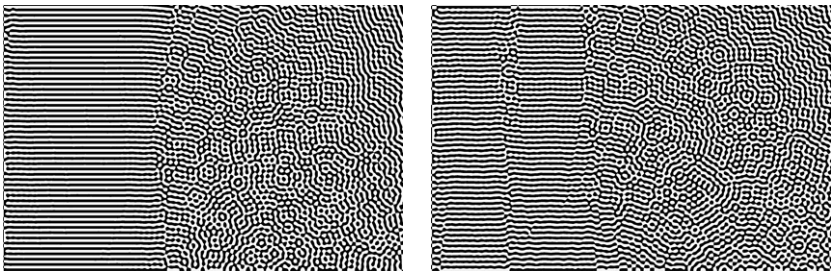

Figure 3. Left: the stadium eigenfunction with $E = 10098.531$. Right: the same but with $E = 10107.147$.

In [3], it was demonstrated that these relations are well fulfilled by random wavefunctions. For completeness, we plot in figure 2 the energy dependence of number of nodal domains computed numerically for true eigenfunctions of a quarter of the stadium close to the 10000th level. This figure confirms that expression (5) agrees well with the results of numerical calculations for chaotic wavefunctions. Still the numerical data present deviations from the expected value (5) larger than it follows from (6). The origin of such large fluctuations is related to the existence of wavefunction scars. For illustration we present in figure 3 pictures of nodal domains corresponding to the two lowest points in figure 2. It is clearly seen that these pictures have two different parts. One shows a regular pattern typical for integrable systems and the other is practically indistinguishable from a random wavefunction as in figure 1. It is evident that quasi-regular regions are related to the bouncing ball scar. The left eigenfunction corresponds to the single excitation of the bouncing ball and the right one demonstrates the double perpendicular excitation. Regions of quasi-integrable behavior have considerably less number of nodal domains than chaotic regions which explains the wide distribution of points in figure 2.

In [3], it was also checked that the distribution of nodal domain areas agrees with the percolation theory prediction

$$n(s) \sim s^{-\tau} \qquad \text{with} \qquad \tau = 187/91 \tag{7}$$

and the fractal dimension of nodal clusters is close to the percolation value

$$D = 91/48. \tag{8}$$

These and other numerical calculations confirm that nodal domains of random wavefunctions (and of chaotic quantum systems) are correctly described by critical percolation theory.

However, this conjecture has an intrinsic difficulty [6]. The standard percolation theory deals with the following situation. One has a lattice with each site (or edge) positive with probability p or negative with probability $1 - p$. The important assumption here is that these probabilities (or concentrations) at different points are independent random variables. For random and chaotic wavefunctions this is not the case. From (3) it follows that

$$\langle \Psi(x, y) \rangle = 0, \tag{9}$$

and the values of the wavefunction at two points are correlated

$$\langle \Psi(\vec{x}_1)\Psi(\vec{x}_2) \rangle = \sigma^2 g(|\vec{x}_1 - \vec{x}_2|), \tag{10}$$

where

$$g(|\vec{x}_1 - \vec{x}_2|) = \sum_{m=-\infty}^{\infty} J_m(kr_1) J_m(kr_2)\, \mathrm{e}^{\mathrm{i}m(\phi_1 - \phi_2)} = J_0(k|\vec{x}_1 - \vec{x}_2|). \tag{11}$$

In mathematical literature one often defines a Gaussian random function without explicit representation as in (1) but by postulating its mean value and variance (cf (9) and (10)), all other correlation functions being determined by the Wick theorem.

It is well known [7] that for a Gaussian random function the probability that its value at two points has the same sign is

$$P(|\vec{x}_1 - \vec{x}_2|) = \frac{1}{2} + \frac{1}{\pi} \arcsin G(|\vec{x}_1 - \vec{x}_2|), \tag{12}$$

where $G(|\vec{x}_1 - \vec{x}_2|)$ is the two-point correlation function divided by its value at 0 so that $G(0) = 1$. Therefore, if a random Gaussian function is positive at a point, at all close-by points it will be also positive with probability close to 1. Only at large distances when $G(r) \to 0$ probabilities of being positive and negative become equal as in the usual percolation. For random wavefunctions $G(r)$ coincides with $g(r)$ in (11) and because

$$J_0(kr) \stackrel{r\to\infty}{\longrightarrow} \sqrt{\frac{2}{\pi kr}} \cos(kr - \pi/4), \tag{13}$$

the random wavefunction correlation function decays quite slowly ($\sim r^{-1/2}$) which may destroy the validity of percolation theory.

One of the purposes of this paper is to show that this is not the case. In section 2 we use so-called Harris' criterion [8] to demonstrate that the oscillating character of the correlation function (11) leads to strong cancelations and even the slow decay as in (13) is sufficient in order that nodal domains of random wavefunctions be in the same universality class as non-correlated percolation. Though Harris' criterion is not mathematically rigorous, it is widely accepted (see e.g. [9]) and its fulfillment is a strong argument in favor of the validity of the conjecture that nodal domains of random wavefunctions in the semiclassical limit $k \to \infty$ are described by critical percolation. In section 3, we generalize this conjecture to level domains, i.e. regions where $\Psi(x, y)$ is bigger (or smaller) than a non-zero value ε. We argue that such domains are also described by percolation theory but this time by the non-critical percolation and the deviation from criticality is proportional to ε.

2. Harris' criterion

Harris proposed his criterion in 1974 [8] to investigate the influence of random defects on the critical behavior of the Ising model. Later it was applied to many other models. In particular, in [9] it was used for correlated percolation and we follow closely this paper.

Let us consider a critical percolation problem where concentrations in different sites, denoted by $\Psi(\vec{x})$, are correlated. The criticality condition means that the mean concentration in each site equals a critical value p^*,

$$\langle \Psi(\vec{x}) \rangle = p^*. \tag{14}$$

Below we assume that the connected part of the two-point correlation function of concentrations depends only on the difference between these points

$$\langle (\Psi(\vec{x}_1) - p^*)(\Psi(\vec{x}_2) - p^*) \rangle = g(|\vec{x}_1 - \vec{x}_2|). \tag{15}$$

Harris' criterion was developed to give a condition on the function $g(r)$ under which correlations are unessential, and critical properties of the problem under consideration are the same as for non-correlated percolation. Harris' argumentation is as follows [8, 9]. Let p_V be the average concentration in a finite volume V,

$$p_V = \frac{1}{V} \sum_{\vec{x} \in V} \Psi(\vec{x}). \tag{16}$$

The mean value of p_V over realizations equals the critical value

$$\langle p_V \rangle = p^*, \tag{17}$$

but for a given realization it differs from it.

Consider the situation when inside the volume V all sites have the same concentration p_V. As $p_V \neq p^*$, it corresponds to a non-critical percolation. From the percolation theory it is known that points are uncorrelated if they are separated by a distance larger than a certain correlation length (the cluster size) ξ:

$$\xi \sim |p_V - p^*|^{-\nu}, \tag{18}$$

where ν is a certain critical index. For the standard two-dimensional percolation $\nu = 4/3$ [5]. When p_V is close to p^* this correlation length is large but finite. Divide the whole space into cells of radius of order of ξ in (18). Different cells can be considered as uncorrelated but inside each cell concentrations are correlated. The natural measure of the importance of correlations is the variance, Δ, of p_V which can be computed from the knowledge of the two-point correlation function (15)

$$\begin{aligned} \Delta \equiv \langle (p_V - p^*)^2 \rangle &= \frac{1}{V^2} \sum_{\vec{x}_1, \vec{x}_2 \in V} \langle (\Psi(\vec{x}_1) - p^*)(\Psi(\vec{x}_2) - p^*) \rangle \\ &\approx \frac{1}{V^2} \int_{\vec{x}_1 \in V} \int_{\vec{x}_2 \in V} g(|\vec{x}_1 - \vec{x}_2|)\, \mathrm{d}\vec{x}_1\, \mathrm{d}\vec{x}_2. \end{aligned} \tag{19}$$

Harris' criterion [8, 9] states that if the variance is small with respect to $|p_V - p^*|^2$,

$$\Delta \ll |p_V - p^*|^2, \tag{20}$$

correlations are unessential and all critical quantities are the same as for the standard uncorrelated percolation.

As the integrant in (19) depends only on the distance between two points one usually simplifies this integral without discussion (cf [9]) by assuming that one point is somewhere

inside V and the second point is on a distance r of the order of the initial cell size so that the double integral in (19) can be rewritten as a simple one

$$\Delta \approx \frac{1}{V}\int_{\vec{x}\in V} g(|\vec{x}|)\,\mathrm{d}\vec{x}. \tag{21}$$

When the d-dimensional cell V is of the radius ξ, equation (21) leads to the following widely used expression:

$$\Delta \approx \frac{1}{\xi^d}\int_0^{\xi} g(r) r^{d-1}\,\mathrm{d}r. \tag{22}$$

If the correlation function decreases as a certain power of the distance

$$g(r) \stackrel{r\to\infty}{\longrightarrow} r^{-\alpha}, \tag{23}$$

one gets

$$\int^{\xi} r^{-\alpha+d-1}\,\mathrm{d}r = \begin{cases} \text{const} & \text{when } \alpha > d \\ \xi^{d-\alpha} & \text{when } \alpha < d. \end{cases} \tag{24}$$

Therefore

$$\Delta \approx \begin{cases} \xi^{-d} \sim |p_V - p^*|^{d\nu} & \text{when } \alpha > d \\ \xi^{-\alpha} \sim |p_V - p^*|^{\alpha\nu} & \text{when } \alpha < d. \end{cases} \tag{25}$$

The comparison with (20) leads to the standard Harris criterion [9] that correlations are unessential provided $\min(\alpha, d)\nu > 2$.

For random wavefunctions $g(r)$ decays as in (13) and because for the two-dimensional percolation $\nu = 4/3$ Harris' criterion seems to indicate that random waves cannot be described by a usual percolation model. But the above considerations are valid only when the correlation function $g(r)$ is positive. When it has oscillations, formulae (21) and (22) cannot be correct as by definition Δ is non-negative, but these expressions may change the sign.

The main point is that the double integral in (19) cannot, in general, be approximated by the simple integral (21).

For random wavefunctions, the correlation function is given by (11)

$$g(|\vec{x}_1 - \vec{x}_2|) \sim J_0(k|\vec{x}_1 - \vec{x}_2|) = \sum_{m=0}^{\infty} J_{|m|}(kr_1) J_{|m|}(kr_2)\,\mathrm{e}^{\mathrm{i}m(\phi_1-\phi_2)}. \tag{26}$$

Assuming for simplicity that region V is a circle, all integrals over polar angles vanish except the one with $m = 0$ and

$$\int_{\vec{x}_1\in V}\int_{\vec{x}_2\in V} g(|\vec{x}_1 - \vec{x}_2|)\,\mathrm{d}\vec{x}_1\,\mathrm{d}\vec{x}_2 = \left(2\pi\int_0^{\xi} J_0(kr) r\,\mathrm{d}r\right)^2. \tag{27}$$

Therefore in this case

$$\Delta \sim \left(\frac{1}{\xi^2}\int_0^{\xi} J_0(kr) r\,\mathrm{d}r\right)^2 \sim \left(\frac{J_1(k\xi)}{\xi}\right)^2 \sim \xi^{-3}. \tag{28}$$

Finally, as $\nu = 4/3$ one concludes that

$$\Delta \sim |p_V - p^*|^4 \ll |p_V - p^*|^2 \tag{29}$$

which means that Harris' criterion is fulfilled and heuristically there is no contradiction between the applicability of the percolation model to nodal domains of random (and chaotic) wavefunctions and slow decay of wavefunction correlations.

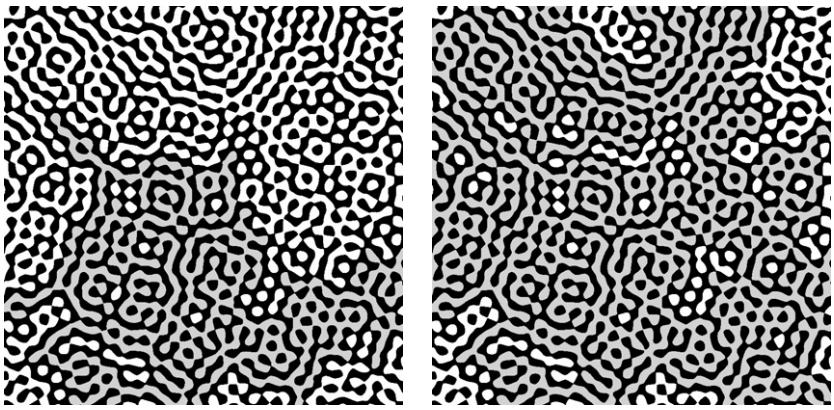

Figure 4. Left: nodal domains of a random wavefunction. Right: level domains of the same function with $\varepsilon = 0.03$. In the both figures the largest connected clusters are highlighted.

3. Level domains and non-critical percolation

The success of percolation-like description of nodal domains of random wavefunctions naturally leads to different generalizations. In particular, it was mentioned in [3] that level domains of random functions can also be described by percolation theory. But contrary to the above case one has to consider not critical but non-critical percolation theory.

Level domains are regions where a function is bigger than a certain value

$$\Psi(x, y) > \varepsilon \tag{30}$$

with $\varepsilon \neq 0$. To understand better how level domains look like we present in figure 4 two pictures. The left one displays nodal domains for one realization of a random wavefunction (3). Boundaries between black and white nodal regions correspond to nodal lines where $\Psi(x, y) = 0$. The right figure represents level domains of the same wavefunction. White (resp. black) regions are domains where $\Psi(x, y)$ is bigger (resp. lower) than 0.03 (in a certain normalization). Boundaries between these regions (called level lines) are solutions of equation $\Psi(x, y) = 0.03$. Both figures look quite similar. To stress their difference the largest connected clusters in both figures are highlighted. In the left figure this cluster connects different boundaries but does not cover the whole region. In contrast, in the right figure the cluster fills practically all allowed space. This behavior is quite reminiscent of percolation transition. At critical concentration there exist an infinite cluster and a large number of finite clusters (cf (7) and (8)). When the concentration is bigger than the critical, one infinite cluster with the dimension equal to the space dimension appears.

To quantify different relations between level domains and non-critical percolation we use one-parameter scaling which is assumed to be valid for non-critical percolation not too far from criticality. According to the scaling conjecture all quantities for non-critical percolation with concentration p are the same as for critical percolation with concentration p^* multiplied by a function depending only on one universal combination of parameters.

For example, the number of cluster of size s in critical case is given by (7). For non-critical percolation it takes the following form:

$$n_s(p) = K_1 s^\tau f(K_2(p - p^*)s^\sigma), \tag{31}$$

where $f(z)$ is a certain function depending on the universal argument

$$z = (p - p^*)s^\sigma. \tag{32}$$

Here τ and σ are universal critical indices. For two-dimensional percolation they are known analytically (see e.g. [5])

$$\tau = \tfrac{187}{91}, \qquad \sigma = \tfrac{36}{91}. \tag{33}$$

In (31) only the constants K_1, K_2 and the threshold value p^* depend on micro details of the problem such as the type of percolation (side or edge percolation) and the form of the lattice (triangular, square, etc). The indices τ, σ and the function $f(z)$ are assumed to be universal (but depend on dimensionality of the problem).

Scaling and universality are very important properties of non-critical percolation theory. We will now check that they are also well fulfilled for level domains of random wavefunctions. First we need a relation between the deviation from criticality, $p-p^*$, and ε in (30). To simplify the formulae below, it is convenient to normalize billiard wavefunctions by the condition

$$\int_A |\Psi(x, y)|^2 \, \mathrm{d}x \, \mathrm{d}y = A, \tag{34}$$

where A is the billiard area.

With this normalization random wavefunctions (3) are Gaussian random functions with $\sigma = 1$ (cf (11)). Therefore, the probability density that $\Psi(x, y) = t$ is

$$P(t) = \frac{1}{\sqrt{2\pi}} \, \mathrm{e}^{-t^2/2} \tag{35}$$

and the probability $P_>(\varepsilon)$ that $\Psi(x, y) > \varepsilon$ takes the form

$$P_>(\varepsilon) = \int_\varepsilon^\infty P(t) \, \mathrm{d}t \approx \frac{1}{2} - \frac{\varepsilon}{\sqrt{2\pi}} + \mathcal{O}(\varepsilon^3). \tag{36}$$

Consequently, for small ε,

$$p - p_c \sim \varepsilon \tag{37}$$

and for level domains the scaling relations as in (31) are valid with the substitution $p - p^* \longrightarrow \varepsilon$.

Consider first the total number of connected level domains. For this quantity there exists no explicit scale as in (31) and the scaling prediction is particular simple. For the non-critical percolation

$$\frac{\bar{n}(p)}{\bar{n}(p^*)} = f(p - p^*), \tag{38}$$

so for level domains it should be

$$\frac{\bar{n}(\varepsilon)}{\bar{n}(0)} = f(K\varepsilon) \tag{39}$$

with a certain constant K and the same function $f(z)$. Here $\bar{n}(p)$ is the mean number of connected clusters for a percolation model with concentration p. $\bar{n}(\varepsilon)$ is the same quantity but for level domains (30). $\bar{n}(0)$ is the mean number of nodal domains given by (5).

In figure 5, there are two curves. The solid line is the ratio (39) computed numerically for random Gaussian functions. Data are well fitted by a quadratic fit $1 + 0.6135\varepsilon + 13.47\varepsilon^2$. The dashed line represents the ratio (38) computed numerically for bond percolation on 110×110 square lattice. Black circles result from the rescaling $p - p^* = \varepsilon/1.62$. It is clearly seen that points for level domains after rescaling agree very well with data from pure percolation which confirms the scaling (38) and (39).

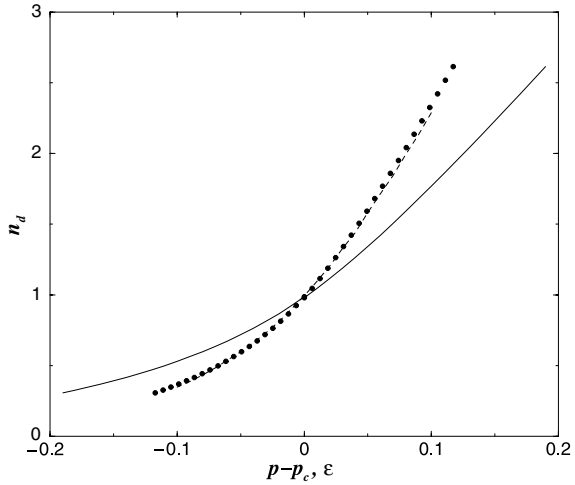

Figure 5. Ratio of the number of clusters to its critical value for level domains of random wavefunctions (solid line) and for non-critical bond percolation on a square lattice. For the first curve the abscissa axis is ε, for the second one it represents $p - p^*$. Black circles are results of rescaling $p - p^* = \varepsilon/1.62$.

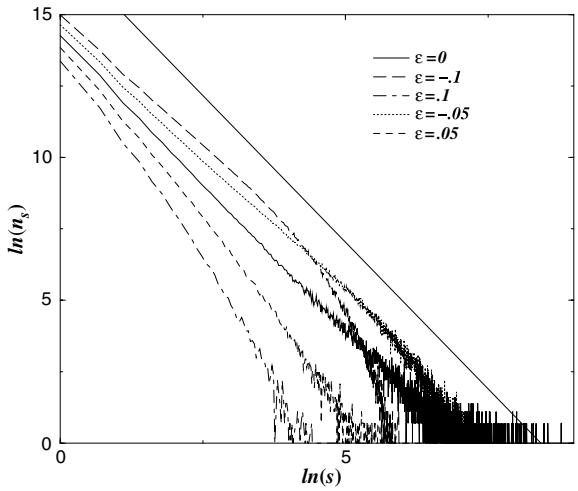

Figure 6. Number of domains of area s for random wavefunctions. Different curves represent results for different values of ε. Straight line has the slope $-187/91$ predicted for nodal domains.

In figures 6 and 7, we present the verification of (31) for level domains of random wavefunctions. In figure 6, the number of clusters with area s is plotted for a few different values of ε and $k = 200$. Curves are clearly different and slopes for $\varepsilon \neq 0$ differ from the critical percolation value (7) predicted for nodal domains. Then we rescale these data using as the abscissa axis the variable $z = \varepsilon s^{\sigma}$ as follows from the one-parameter scaling. The resulting points are concentrated close to each other and on the left of figure 7 the solid line indicates the average of all such calculations which supports the existence of scaling for level domains. In the same figure two more curves are presented. The dashed line represents the

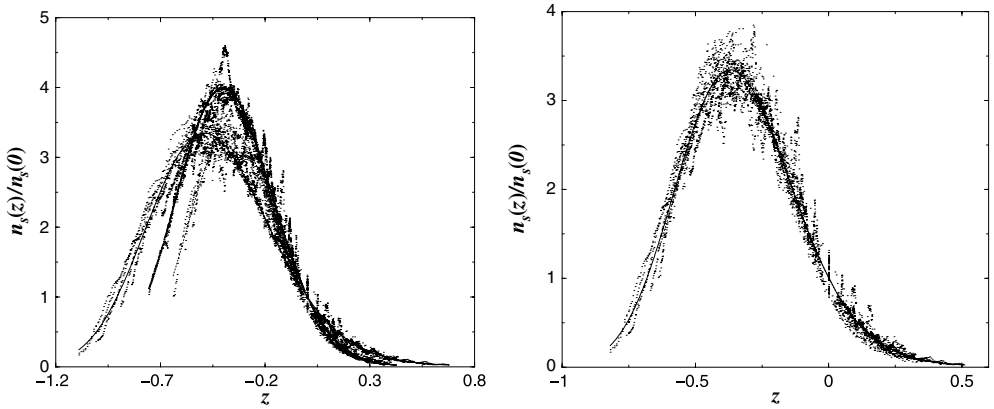

Figure 7. Left: scaling relations. Solid line—random wavefunctions with $k = 200$. Dashed line—bond percolation on a square 110×110 lattice. Dotted line—the same but on a 50×50 lattice. Right: rescaling of dotted curve by $p - p^* = \varepsilon/1.33$.

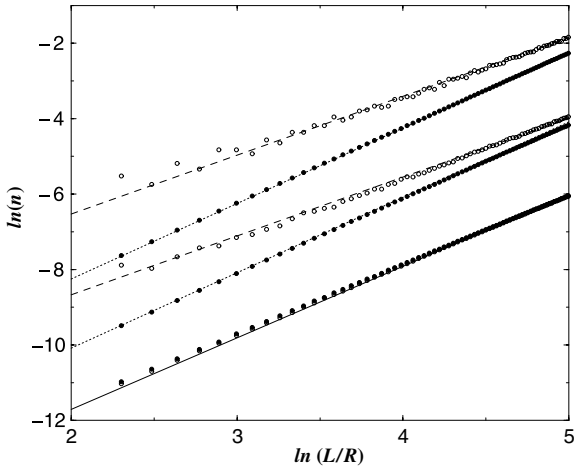

Figure 8. Fractal dimension of level domains. From bottom to top: $\varepsilon = 0$, $\varepsilon = 0.1$, and $\varepsilon = 0.2$. For all curves $k = 200$. The slope of the solid line is $91/48$, of the dotted line is 2, of the dashed line it is 1.56. Black circles represent domains where $\Psi(x, y) < \varepsilon$ and open ones regions with $\Psi(x, y) > \varepsilon$.

results of similar calculations but for the non-critical bond percolation on a square 110×110 lattice and the dotted line is the same but on a square lattice with 50×50 sites. For these curves the z variable is defined as in (31), namely, $z = (p - p^*)s^\sigma$. Once more the scaling relation is well satisfied. Finally, on the right part of figure 8 it is shown that after the rescaling $p - p^* = \varepsilon/1.33$, the curve of level domains practically coincides with the one for non-critical percolation on a square $50\times$ lattice.

Finally, in figure 8 the results of numerical calculations for fractal dimensions of clusters are presented. According to percolation theory (see e.g. [5]) the dominant cluster has the dimension 2 but the other ones have the fractal dimension equal approximately to 1.56. Note that for critical percolation (and the nodal domains) the fractal dimension of clusters is $91/48$.

4. Conclusion

The conjecture that random (and chaotic) wavefunctions in the semiclassical limit can be described by percolation theory is analyzed. We demonstrate that, though wave correlations decay slowly, a careful application of Harris' criterion suggests that nodal domains of random wavefunctions are indeed in the same universality class as the critical non-correlated percolation. We also present arguments in favor that the non-critical percolation is useful in the investigation of non-zero level domains of random wavefunctions. Let us mention that in [10] it is shown that nodal lines of random wavefunctions are close to SLE_6 curves which describe boundaries of critical percolation clusters. Recently in [11] it has been proved the existence of the limit (5) (but not its value) for random spherical harmonics. All these results confirm the above conjecture. It is of interest to find a rigorous proof of it.

Acknowledgments

EB is grateful to G Foltin, U Smilansky, and J Keating for stimulating discussions and to O Bohigas for a critical reading of the manuscript. He is also thankful to RM Ziff for pointing out [4] and to a referee for indicating [11].

References

[1] Berry M V 1977 Regular and irregular semiclassical wave functions *J. Phys. A: Math. Gen.* **10** 2083
[2] Blum G, Gnutzmann S and Smilansky U 2002 Nodal domains statistics: a criterion for quantum chaos *Phys. Rev. Lett.* **88** 11410
[3] Bogomolny E and Schmit C 2002 Percolation model for nodal domains of chaotic wave functions *Phys. Rev. Lett.* **88** 114102
[4] Ziff R M, Finch S R and Adamchik V S 1997 Universality of finite—size corrections to the number of critical percolation clusters *Phys. Rev. Lett.* **79** 3447
[5] Stauffer D and Aharony A 1994 *Introduction to Percolation Theory* (London: Taylor and Francis)
[6] Foltin G, Gnutzmann S and Smilansky 2004 The morphology of nodal lines—random vawes versus percolation *J. Phys. A: Math. Gen.* **37** 11363
[7] Rice S O 1944 Mathematical analysis of random noise *Bell Syst. Tech. J.* **23** 282
Rice S O 1945 Mathematical analysis of random noise—conclusion *Bell Syst. Tech. J.* **24** 46
[8] Harris A B 1974 Effect of random defects on the critical behavior of Ising model *J. Phys. C: Solid State Phys.* **7** 167
[9] Weinrib A 1984 Long-range correlated percolation *Phys. Rev.* B **29** 387
[10] Bogomolny E, Schmit C and Dubertrand R 2007 SLE description of the nodal lines of random wavefunctions *J. Phys. A: Math. Theor.* **48** 381
[11] Nazarov F and Sodin M 2007 On the number of nodal domains of random spherical harmonics (*Preprint* math-ph:0706.2409)